APPLICATION OF THE THEORY OF THERMAL SIMILARITY TO THE STUDY OF THE ELECTRIC ARC

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One of the basic difficulties in developing a theory to describe an electric arc burning under conditions of natural convection is the need to take convective heat transfer into account in the energy balance of the arc. This makes it necessary to solve not only the energy balance equation but also the equation of motion of the gas [1, 2]. Owing to the mathematical difficulties, this rigorous method of solving the problem is scarcely feasible. It is therefore of interest to study the convective heat transfer in an arc column using the relations of the theory of thermal similarity, as has already been done for convection from solids [3, 4]. A number of authors [5, 6] have made progress in this direction on the assumption that the arc column is an isothermal channel of radius R and temperature T, heat from which is removed by convection. Since the initial assumption concerning a uniform isothermal channel and the additional assumptions about the constancy of the quantities T [5] and $I/\pi R^2$ [6] at different currents I are too rough an approximation to reality, the authors of [5, 6] do not answer the question of the extent to which application of the relations of the theory of thermal similarity is justified in the case of an arc discharge.

A. M. Zalesskii [7] has objected to the use of the theory of thermal similarity. However, his calculations, which alledgedly prove the unsuitability of the theory in the case of convection from an arc discharge, are not completely convincing, since they contain a number of quite arbitrary assumptions. Below it is shown that, given a more rigorous approach to the solution of the problem, the use of the theory of thermal similarity leads to results that are in satisfactory agreement with experiment.

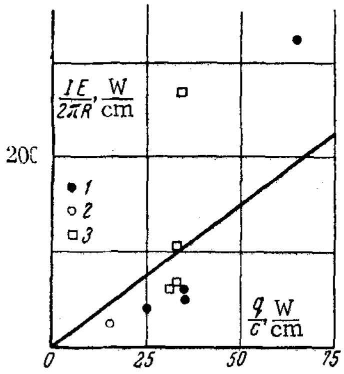

Fig. 1. Relations IE/2πR vs. q/c: 1) carbon arc in air [1, 10, 15]; 2) sodium arc in air [16]; 3) arc in mixture of 90% Ar + 10% H [17].

An experimental investigation of convection in an arc discharge [8, 9] shows that the convective losses from the inner zone of the arc are negligibly small, since the direction of the gas flow velocity V in this zone is parallel to the isothermal surfaces of the arc column [1, 9, 10], as a result of which the convective heat flux $\rho c_p V \operatorname{grad} T$ is almost zero. Of course, in an arc discharge, as distinct from a solid, there is no sharp boundary surface, from which convective heat transfer takes place. However, in a first approximation the differentiation of an inner zone and a convection zone is perfectly realistic.

If it is assumed that the inner zone of a vertical electric arc is cylindrically symmetrical, the energy balance equation for this zone will have the form [1]

$$\frac{1}{r}\frac{d}{dr}\left(\varkappa r\frac{dT}{dr}\right) - p + \sigma E^2 = 0. \tag{1}$$

Here T is the plasma temperature at a distance r from the axes of the arc, $\varkappa$ and σ are the thermal conductivity and electrical conductivity of the plasma, E is the longitudinal component of the electric field strength, p is the energy radiated in unit time from unit volume of the plasma. One of the boundary conditions for Eq. (1) will be

$$dT/dr = 0 \quad \text{at} \quad r = 0\,. \tag{2}$$

If, as previously assumed [11], we consider that the boundary between the inner zone and the convection zone is a certain isothermal surface of temperature T^*, then the second boundary condition for (1) can be written in the form [11]

$$\varkappa dT/dr = -q \quad \text{at} \quad T = T^*\,. \tag{3}$$

The quantity q is given by the relation

$$q = \alpha(T)(T - T_\infty). \tag{4}$$

Here T_∞ is the ambient temperature. The heat transfer coefficient α is found using the relation between the dimensionless Nusselt N, Grashof G, and Prandtl P numbers known from the theory of thermal similarity [3, 4, 11]

$$N = CG^{1/3}P^{1/3}\,. \tag{5}$$

Expanding these criteria, we determined the value of α, and then that of q

$$q = Cg^{1/3}\beta_c^{1/3}\varkappa_c^{1/2}c_{pc}^{1/2}\rho_c^{2/3}\eta_c^{-1/6}(T - T_\infty)^{4/3}.$$

Here g is the free-fall acceleration; β, $\varkappa$, c_p, ρ and η are, respectively, the coefficient of volume expansion, thermal conductivity, specific heat at constant pressure, density, and dynamic viscosity of the gas. The subscript c indicates that the mean integral value of the quantity is taken [7]

$$\varphi_c = \frac{1}{T - T_\infty}\int\limits_{T_\infty}^{T} \varphi dT.$$

Since the electrical energy is almost all released within the limits of the inner zone, we get the equality [11]

$$IE - L = 2\pi R_q \qquad \left(L = \int\limits_0^R p2\pi r dr\right), \quad R = r \quad \text{at} \quad T = T^*. \tag{7}$$

Here L is the total radiation energy. Relations (1)-(3) and (7) are sufficient to solve the problem if the value of T^* is known.

We previously assumed [11] that for an arc in air T^* = const (≈ 3000°K) at different currents. In this case $\varkappa$ and q in relation (3), being single-valued functions of the temperature T^*, will also be constant quantities, which enables us to write (3) in the form [11]

$$dT / dr = \text{const} \quad \text{at} \quad T = T^*. \tag{8}$$

Putting L = 0 [12, 13], since $q(T^*)$ = const at T^* = const, from Eq. (7) we can obtain

$$IE / R = \text{const}. \tag{9}$$

Relations (8), (9), which express the conditions of nondependence of the quantities $(dT/dr)_{T=T^*}$ and IE/R on the current, are in good agreement with experiment in the case of an arc in air I = 2-20 amps and T^* = 3000°K [11]. However, a shortcoming of this approach to the evaluation of the quantity T^* is the comparatively small interval of currents in which relations (8), (9) hold true, and also the necessity of determing T^* experimentally for each new plasma composition. It therefore proved desirable to find some more general method of determining the quantity T^*. From a consideration of the experimental data on the temperature field and the field of gas flow velocities for a carbon arc in air at a current of 10 amps [9, 10] and 200 amps [1] it was found that in the first case at an axial temperature T_0 = 7000°K convective heat transfer begins in the region 3000-4000°K, and in the second case at T_0 = 10 900°K the region of convection begins at 5000-6000°K, i. e., one gets the approximate equality

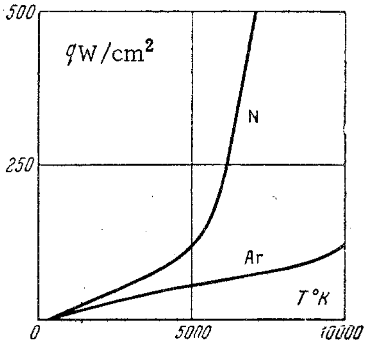

Fig. 2. The relation q = q(T).

$$T^* = {}^1/_2 T_0. \tag{10}$$

From the radial temperature distribution curves presented in [1, 10] it is also apparent that the slope of the curve T(r) changes markedly at temperatures of the order of $T_0/2$, becoming considerably flatter, which indicates the presence of convective heat transfer. Since for a small change in current the axial temperature T_0 remains practically constant, it follows from (10) that T^* = const, i. e., our previous assumption concerning the nondependence of T^* on current is obtained as a special case of Eq. (10).

In order to establish the generality of relation (10), we checked its applicability for arc discharges with different plasma compositions. Since we could not find other data on convection in arcs, apart from the reference already mentioned, we verified relation (10) by an indirect method. If it is assumed that radiative energy transfer is an inconsiderable fraction of the energy balance of the arc* (i. e., if we put L = 0), then from (7) we have

$${}^1/_2 IE / \pi R = q. \tag{11}$$

This equation must be satisfied in the case R = r at $T_0/2$. On the basis of existing experimental data on several types of arc discharge [1, 10, 15-17] we constructed a graph of the relation IE/2πR versus q/c (the data needed to calculate q/c were taken from [18-23]). As may be seen from Fig. 1, in the first approximation 1/2IE/πR ~ q/c, which indi-

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*Evidently, this assumption will be satisfactory enough for an arc discharge in nitrogen and air [12, 13], but very approximate for an arc in inert gases [14].

cates satisfaction of relation (11) obtained with the aid of the theory of thermal similarity for $T^* = 1/2T_0$. The considerable scatter of the points on the graph is due to the following reasons.

(1) Relations (10) and (11) are approximate.

(2) The proposed approach to arc column theory disregards certain differences in the experimental conditions (size and spacing of electrodes).

(3) Errors in determining the values of E, T_0, and R.

(4) Errors in calculating values of q/c.

Figure 1 may be used to determine the constant c in relations (5), (6). The value c ~ 3 obtained differs considerably from the value c = 0.135 [4], which corresponds to the case of turbulent convection from a verticle solid cylinder. It should be noted that the value 0.135 was obtained at fairly low, as compared with the arc, values of the temperature. Moreover, it is clear that the relations of the theory of thermal similarity can only reflect the general features of convective heat transfer in an arc discharge, so that it is impossible to expect perfect agreement in relation to the value of the constant.

Thus, introducing into the calculations the single empirical value c = 3 enables one, with the aid of relations (1)-(3), (6), and (10), to uniquely determine the parameters of an electric arc column for any composition of the arc plasma. Since the temperature T_0 in (10) is unknown, Eq. (1) is solved by the method of successive approximations. As calculations show, the approximation process converges rapidly.

In solving Eq. (1) it is convenient to introduce the new variable

$$S = \int_{T^*}^{T} \varkappa dT \tag{12}$$

and to approximate the complex function σ(S) with two segments of a straight line

$$\sigma = \begin{cases} a^2 (S - S_1) & \text{for } S_0 \geqslant S \geqslant S_1 \\ 0 & \text{for } S_1 \geqslant S \geqslant 0 \end{cases}$$
$$(S_0 = S(T_0)). \tag{13}$$

Here a^2 and S_1 are constant coefficients. This approximation, while not introducing any serious error, makes it possible to solve Eq. (1) in analytic form in the case p = 0 [23, 24].

Using this method we calculated the parameters of an arc discharge in nitrogen and in argon at currents from 1 to 300 amps. As a preliminary step we computed the value of q over a wide range of temperatures (Fig. 2). The electrical conductivity of the plasma, which enters into Eq. (1), was calculated in accordance with [25, 26], the cross section for elastic scattering of electrons at Ar atoms being taken from [26], at N atoms from [27], and at N_2 molecules from [28, 29]. The thermal conductivity of argon at temperatures up to 8000°K was computed from the data [20], and from 8000 to 15 000°K it was taken from [30]. For the thermal conductivity of nitrogen in the region 1000-7000°K we took the calculated curve from [19], and in the region 7000-15 000°K the experimental curve from [13] (in the range 5500-7000°K the two curves coincide correct to 10%).

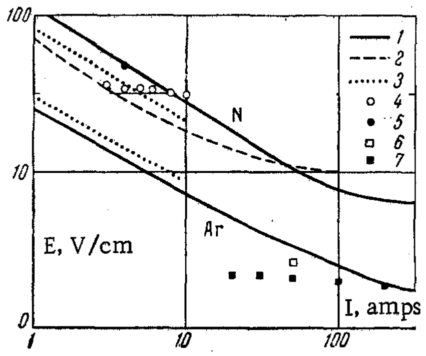

Fig. 3. Current dependence of electric field strength: 1) theoretical calculation, 2) King's data [31], 3) Suits' data [32], 4 and 5) data of Somers and Smit [33], 6) Kolesnikov's data [17], 7) Goldman's converted data [34].

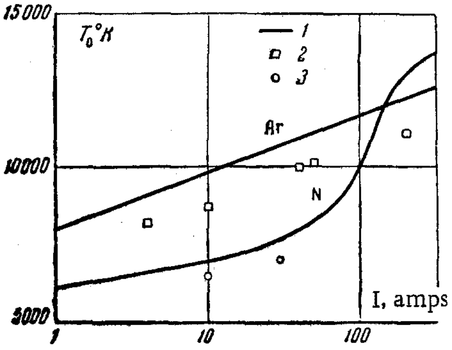

Fig. 4. Current dependence of temperature at arc axes: 1) theoretical calculation, 2) arc in mixture of 90% Ar + 10% H [17], 3) arc in nitrogen [36].

The current dependence of the field strength for arcs in nitrogen and argon is presented to a logarithmic scale in Fig. 3, from which it is clear that the theoretical and experimental curves E(I) are similar in form. At small currents the theoretical curve corresponds to the known empirical relation $E \sim I^{-n}$ [32, 35], very close correspondence being observed even in the value of the quantity n (calculations give $n = 0.62$ for N and $n = 0.56$ for Ar, the experimental values [32] being 0.60 and 0.54, respectively). As the current increases the theoretical curve tends to the form E = const, which is also in accordance with experiment. At the same time, it is clear from Fig. 3 that in a number of cases there is a considerable discrepancy in the values of the theoretical and experimental field strengths. The lower experimental values of E for an arc in nitrogen at currents from I = 1 amp to I = 10 amps may be conditioned by the easily ionized impurities in the electrodes. Illuminating in this respect are the data of Somers and Smit [33], who, using electrodes of different purity, obtained different values of E (solid and open circles in Fig. 3). Since the dependence of V on l is not strictly linear, the values of E for an arc in argon lying below the theoretical curve were also found to be unreliable. We obtained these values by converting Goldman's data [34] in accordance with the formula $E = (V_1 - V_2)/(l_1 - l_2)$, where V_1 is the voltage between the arc electrodes at an arc gap length $l_1 = 15$ mm, and V_2 is the voltage at $l_2 = 10$ mm.

Figure 4 presents the results of calculating the temperature at the axes of the discharge T_0 for different currents. For an arc in argon, within the limits of accuracy of the calculations, the relation $T_0 = 8000 + 1800 \lg I$, where T_0 is in °K, and I is in amps, is satisfied. In the case of nitrogen the relation $T_0(I)$ has a similar form in the interval I = 1-10 amps, but then the temperature begins to increase sharply with increase in current. A comparison of the curves $T_0(I)$ and $\varkappa(T)$ shows that the curve $T_0(I)$ is steepest in the temperature interval 10 000-11 000°K, which corresponds to the region of the minimum in the temperature dependence of the thermal conductivity of nitrogen [13]. A comparison of the theoretical results with experiment is rendered somewhat difficult by the scarcity of suitable experimental data, since only the results of temperature measurements on sufficiently long arcs may be used to verify the theory.* The results of Kolesnikov's measurements [17] for an arc in argon with hydrogen as an impurity, which satisfy this last condition, are presented in Fig. 4. On the whole, the experimental points are in good agreement with the theoretical dependence of T_0 or I; however, several lie below the theoretical curve, which may be due to the increase in the thermal conductivity of the argon plasma as a result of the presence of hydrogen. For arcs in nitrogen we have only the results obtained by Edels and Whittaker [36] by the shock wave method at currents of 10 and 30 amps. The values of the temperature they obtained lie below the theoretical values at 500-600°K, which is perfectly natural, since the measuring method employed gives a temperature that to some extent is averaged over the cross section of the column (and not T_0).

Thus, in conclusion it may be stated that using the theory of thermal similarity to take account of convective heat transfer in an arc discharge enables one to compute, with satisfactory accuracy, the discharge parameters for arc plasmas of different composition and currents from 1 to 300 amps.

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*The given limitation is due to the fact that in the proposed method of calculating the column parameters the effect of the electrodes is neglected. It may be roughly assumed that the arc column is independent of the electrodes if the relation $l > 0.2\,I$, where l is in mm and I is in amps, is satisfied.

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20 June 1964 Irkutsk